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BASIC ENGINEERING MATHEMATICS

JOHN BIRD

SIXTH EDITION

Basic Engineering Mathematics

Why is knowledge of mathematics important in engineering?

A career in any engineering or scientific field will require both basic and advanced mathematics. Without mathematics to determine principles, calculate dimensions and limits, explore variations, prove concepts, and so on, there would be no mobile telephones, televisions, stereo systems, video games, microwave ovens, computers, or virtually anything electronic. There would be no bridges, tunnels, roads, skyscrapers, automobiles, ships, planes, rockets or most things mechanical. There would be no metals beyond the common ones, such as iron and copper, no plastics, no synthetics. In fact, society would most certainly be less advanced without the use of mathematics throughout the centuries and into the future.

Electrical engineers require mathematics to design, develop, test, or supervise the manufacturing and installation of electrical equipment, components, or systems for commercial, industrial, military, or scientific use.

Mechanical engineers require mathematics to perform engineering duties in planning and designing tools, engines, machines, and other mechanically functioning equipment; they oversee installation, operation, maintenance, and repair of such equipment as centralised heat, gas, water, and steam systems.

Aerospace engineers require mathematics to perform a variety of engineering work in designing, constructing, and testing aircraft, missiles, and spacecraft; they conduct basic and applied research to evaluate adaptability of materials and equipment to aircraft design and manufacture and recommend improvements in testing equipment and techniques.

Nuclear engineers require mathematics to conduct research on nuclear engineering problems or apply principles and theory of nuclear science to problems concerned with release, control, and utilisation of nuclear energy and nuclear waste disposal.

Petroleum engineers require mathematics to devise methods to improve oil and gas well production and determine the need for new or modified tool designs; they oversee drilling and offer technical advice to achieve economical and satisfactory progress.

Industrial engineers require mathematics to design, develop, test, and evaluate integrated systems for managing industrial production processes, including human work factors, quality control, inventory control, logistics and material flow, cost analysis, and production co-ordination.

Environmental engineers require mathematics to design, plan, or perform engineering duties in the prevention, control, and remediation of environmental health hazards, using various engineering disciplines; their work may include waste treatment, site remediation, or pollution control technology.

Civil engineers require mathematics at all levels in civil engineering – structural engineering, hydraulics and geotechnical engineering are all fields that employ mathematical tools such as differential equations, tensor analysis, field theory, numerical methods and operations research.

Knowledge of mathematics is therefore needed by each of the engineering disciplines listed above.

It is intended that this text – *Basic Engineering Mathematics* – will provide a step by step approach to learning all the early, fundamental mathematics needed for your future engineering studies.

To Sue

Basic Engineering Mathematics

Sixth Edition

John Bird, BSc (Hons), CEng, CMath, CSci, FIMA, FIET, FCollT

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Preface

Basic Engineering Mathematics 6th Edition introduces and then consolidates basic mathematical principles and promotes awareness of mathematical concepts for students needing a broad base for further vocational studies. In this sixth edition, new material has been added to some of the chapters, together with around 40 extra practical problems interspersed throughout the text. The four chapters only available on the website in the previous edition have been included in this edition. In addition, some multiple choice questions have been included to add interest to the learning.

The text covers:

- (i) **Basic mathematics** for a wide range of introductory/access/foundation mathematics courses
- (ii) **'Mathematics for Engineering Technicians'** for BTEC First NQF Level 2; ***chapters 1 to 12, 16 to 18, 20, 21, 23, and 25 to 27 are needed for this module***.
- (iii) The mandatory **'Mathematics for Technicians'** for BTEC National Certificate and National Diploma in Engineering, NQF Level 3; ***chapters 7 to 10, 14 to 17, 19, 20 to 23, 25 to 27, 31, 32, 34 and 35 are needed for this module. In addition, chapters 1 to 6, 11 and 12 are helpful revision for this module***.
- (iv) **GCSE revision**, and for similar mathematics courses in English-speaking countries worldwide.

Basic Engineering Mathematics 6th Edition provides a lead into *Engineering Mathematics 7th Edition*.

Each topic considered in the text is presented in a way that assumes in the reader little previous knowledge of that topic.

Theory is introduced in each chapter by a brief outline of essential theory, definitions, formulae, laws and procedures. However, these are kept to a minimum, for problem solving is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then solving similar problems themselves.

This textbook contains some **750 worked problems**, followed by over **1600 further problems** (all with answers – at the end of the book). The further problems are contained within **161 Practice Exercises**; each Practice Exercise follows on directly from the relevant section of work. Fully worked solutions to all 1600 problems have been made freely available to all via the website – see below. **420 line diagrams** enhance the understanding of the theory. Where at all possible the problems mirror potential practical situations found in engineering and science.

At regular intervals throughout the text are **15 Revision Tests** to check understanding. For example, Revision Test 1 covers material contained in chapters 1 and 2, Revision Test 2 covers the material contained in chapters 3 to 5, and so on. These Revision Tests do not have answers given since it is envisaged that lecturers/instructors could set the Tests for students to attempt as part of their course structure. Lecturers/instructors may obtain a complimentary set of solutions of the Revision Tests in an **Instructor's Manual** available from the publishers via the internet – see below.

At the end of the book a list of relevant **formulae** contained within the text is included for convenience of reference.

'Learning by example' is at the heart of *Basic Engineering Mathematics 6th Edition*.

JOHN BIRD
Defence College of Technical Training,
HMS Sultan,
formerly of University of Portsmouth
and Highbury College, Portsmouth

John Bird is the former Head of Applied Electronics in the Faculty of Technology at Highbury College, Portsmouth, UK. More recently, he has combined freelance lecturing at the University of Portsmouth, with Examiner responsibilities for Advanced Mathematics with City and Guilds, and examining for the International Baccalaureate Organization. He is the author of some 125 textbooks on engineering and mathematical subjects, with worldwide sales of 1 million copies. He is currently a Senior Training Provider at the Defence School of Marine Engineering in the Defence College of Technical Training at HMS *Sultan*, Gosport, Hampshire, UK.

Free Web downloads

For students

1. **Full solutions** to the 1600 questions contained in the 161 Practice Exercises
2. Download **Multiple choice questions and answer sheet**
3. **List of Essential Formulae**
4. **Famous Engineers/Scientists** – From time to time in the text, 16 famous mathematicians/engineers are referred to and emphasised with an asterisk*. Background information on each of these is available via the website. Mathematicians/engineers involved are: **Boyle, Celsius, Charles, Descartes, Faraday, Henry, Hertz, Hooke, Kirchhoff, Leibniz, Napier, Newton, Ohm, Pythagoras, Simpson and Young.**

For instructors/lecturers

1. **Full solutions** to the 1600 questions contained in the 161 Practice Exercises
2. **Full solutions** and marking scheme to each of the **15 Revision Tests** – named as **Instructors Manual**
3. **Revision Tests** – available to run off to be given to students
4. Download **Multiple choice questions and answer sheet**
5. **List of Essential Formulae**
6. **Illustrations** – all 420 available on PowerPoint
7. **Famous Engineers/Scientists** – 16 are mentioned in the text, as listed above

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Chapter 1

Basic arithmetic

Why it is important to understand: **Basic arithmetic**

Being numerate, i.e. having an ability to add, subtract, multiply and divide whole numbers with some confidence, goes a long way towards helping you become competent at mathematics. Of course electronic calculators are a marvellous aid to the quite complicated calculations often required in engineering; however, having a feel for numbers 'in our head' can be invaluable when estimating. Do not spend too much time on this chapter because we deal with the calculator later; however, try to have some idea how to do quick calculations in the absence of a calculator. You will feel more confident in dealing with numbers and calculations if you can do this.

At the end of this chapter, you should be able to:

- understand positive and negative integers
- add and subtract integers
- multiply and divide two integers
- multiply numbers up to 12 × 12 by rote
- determine the highest common factor from a set of numbers
- determine the lowest common multiple from a set of numbers
- appreciate the order of operation when evaluating expressions
- understand the use of brackets in expressions
- evaluate expressions containing $+, -, \times, \div$ and brackets

1.1 Introduction

Whole numbers are simply the numbers 0, 1, 2, 3, 4, 5, ... (and so on). **Integers** are like whole numbers, but they also include negative numbers. $+3, +5$ and $+72$ are examples of positive integers; $-13, -6$ and -51 are examples of negative integers. Between positive and negative integers is the number 0 which is neither positive nor negative.

The four basic arithmetic operators are add (+), subtract (−), multiply (×) and divide (÷).

It is assumed that adding, subtracting, multiplying and dividing reasonably small numbers can be achieved without a calculator. However, if revision of this area is needed then some worked problems are included in the following sections.

When **unlike signs** occur together in a calculation, the overall sign is **negative**. For example,

$$3 + (-4) = 3 + -4 = 3 - 4 = \mathbf{-1}$$

and

$$(+5) \times (-2) = \mathbf{-10}$$

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Like signs together give an overall **positive sign**. For example,

$$3 - (-4) = 3 - -4 = 3 + 4 = \mathbf{7}$$

and

$$(-6) \times (-4) = \mathbf{+24}$$

1.2 Revision of addition and subtraction

You can probably already add two or more numbers together and subtract one number from another. However, if you need revision then the following worked problems should be helpful.

Problem 1. Determine 735 + 167

$$\begin{array}{rr} & \mathbf{H\ T\ U} \\ & 7\ \ 3\ \ 5 \\ + & 1\ \ 6\ \ 7 \\ \hline & 9\ \ 0\ \ 2 \\ \hline & 1\ \ 1\ \ \ \ \end{array}$$

(i) 5 + 7 = 12. Place 2 in units (U) column. Carry 1 in the tens (T) column.

(ii) 3 + 6 + 1 (carried) = 10. Place the 0 in the tens column. Carry the 1 in the hundreds (H) column.

(iii) 7 + 1 + 1 (carried) = 9. Place the 9 in the hundreds column.

Hence, **735 + 167 = 902**

Problem 2. Determine 632 − 369

$$\begin{array}{rr} & \mathbf{H\ T\ U} \\ & 6\ \ 3\ \ 2 \\ - & 3\ \ 6\ \ 9 \\ \hline & 2\ \ 6\ \ 3 \\ \hline \end{array}$$

(i) 2 − 9 is not possible; therefore change one ten into ten units (leaving 2 in the tens column). In the units column, this gives us 12 − 9 = 3

(ii) Place 3 in the units column.

(iii) 2 − 6 is not possible; therefore change one hundred into ten tens (leaving 5 in the hundreds column). In the tens column, this gives us 12 − 6 = 6

(iv) Place the 6 in the tens column.

(v) 5 − 3 = 2

(vi) Place the 2 in the hundreds column.

Hence, **632 − 369 = 263**

Problem 3. Add 27, −74, 81 and −19

This problem is written as 27 − 74 + 81 − 19.

Adding the positive integers:

$$\begin{array}{r} 27 \\ 81 \\ \hline \end{array}$$

Sum of positive integers is

$$\begin{array}{r} 108 \\ \hline \end{array}$$

Adding the negative integers:

$$\begin{array}{r} 74 \\ 19 \\ \hline \end{array}$$

Sum of negative integers is

$$\begin{array}{r} 93 \\ \hline \end{array}$$

Taking the sum of the negative integers from the sum of the positive integers gives

$$\begin{array}{r} 108 \\ -93 \\ \hline 15 \\ \hline \end{array}$$

Thus, **27 − 74 + 81 − 19 = 15**

Problem 4. Subtract −74 from 377

This problem is written as 377 − −74. Like signs together give an overall positive sign, hence

$$377 - -74 = 377 + 74 \qquad \begin{array}{rr} & 3\,7\,7 \\ + & 7\,4 \\ \hline & 4\,5\,1 \\ \hline \end{array}$$

Thus, **377 − −74 = 451**

Problem 5. Subtract 243 from 126

The problem is 126 − 243. When the second number is larger than the first, take the smaller number from the larger and make the result negative. Thus,

$$126 - 243 = -(243 - 126) \qquad \begin{array}{rr} & 2\,4\,3 \\ - & 1\,2\,6 \\ \hline & 1\,1\,7 \\ \hline \end{array}$$

Thus, **126 − 243 = −117**

Problem 6. Subtract 318 from −269

The problem is −269 − 318. The sum of the negative integers is

$$\begin{array}{r} 269 \\ +318 \\ \hline 587 \\ \hline \end{array}$$

Thus, **−269 − 318 = −587**

Now try the following Practice Exercise

Practice Exercise 1 Further problems on addition and subtraction (answers on page 422)

In Problems 1–15, determine the values of the expressions given, without using a calculator.

1. 67 kg − 82 kg + 34 kg
2. 73 m − 57 m
3. 851 mm − 372 mm
4. 124 − 273 + 481 − 398
5. £927 − £114 + £182 − £183 − £247
6. 647 − 872
7. 2417 − 487 + 2424 − 1778 − 4712
8. −38419 − 2177 + 2440 − 799 + 2834
9. £2715 − £18250 + £11471 − £1509 + £113274
10. 47 + (−74) − (−23)
11. 813 − (−674)
12. 3151 − (−2763)
13. 4872 g − 4683 g
14. −23148 − 47724
15. $53774 − $38441
16. Calculate the diameter d and dimensions A and B for the template shown in Fig. 1.1. All dimensions are in millimetres.

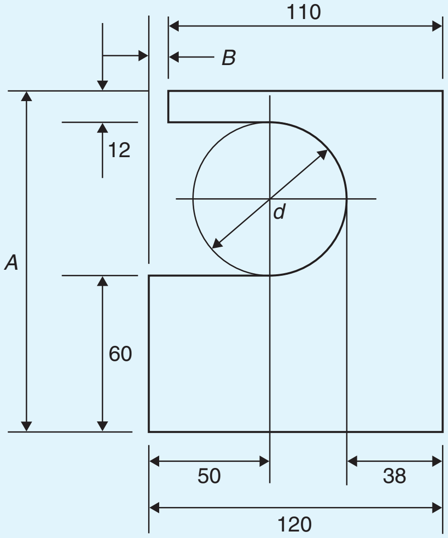

Figure 1.1

1.3 Revision of multiplication and division

You can probably already multiply two numbers together and divide one number by another. However, if you need a revision then the following worked problems should be helpful.

Problem 7. Determine 86 × 7

$$\begin{array}{rrr} \mathbf{H} & \mathbf{T} & \mathbf{U} \\ & 8 & 6 \\ \times & & 7 \\ \hline 6 & 0 & 2 \\ \hline & 4 & \end{array}$$

(i) 7 × 6 = 42. Place the 2 in the units (U) column and ‘carry’ the 4 into the tens (T) column.

(ii) 7 × 8 = 56; 56 + 4 (carried) = 60. Place the 0 in the tens column and the 6 in the hundreds (H) column.

Hence, 86 × 7 = 602
A good grasp of **multiplication tables** is needed when multiplying such numbers; a reminder of the multiplication table up to 12 × 12 is shown below. Confidence with handling numbers will be greatly improved if this table is memorised.

Multiplication table

×	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Problem 8. Determine 764 × 38

$$\begin{array}{r} 764 \\ \times \quad 38 \\ \hline 6112 \\ 22920 \\ \hline 29032 \\ \hline \end{array}$$

(i) $8 \times 4 = 32$. Place the 2 in the units column and carry 3 into the tens column.

(ii) $8 \times 6 = 48$; $48 + 3$ (carried) $= 51$. Place the 1 in the tens column and carry the 5 into the hundreds column.

(iii) $8 \times 7 = 56$; $56 + 5$ (carried) $= 61$. Place 1 in the hundreds column and 6 in the thousands column.

(iv) Place 0 in the units column under the 2

(v) $3 \times 4 = 12$. Place the 2 in the tens column and carry 1 into the hundreds column.

(vi) $3 \times 6 = 18$; $18 + 1$ (carried) $= 19$. Place the 9 in the hundreds column and carry the 1 into the thousands column.

(vii) $3 \times 7 = 21$; $21 + 1$ (carried) $= 22$. Place 2 in the thousands column and 2 in the ten thousands column.

(viii) $6112 + 22920 = 29032$

Hence, **764 × 38 = 29032**

Again, knowing multiplication tables is rather important when multiplying such numbers.

It is appreciated, of course, that such a multiplication can, and probably will, be performed using a **calculator**. However, there are times when a calculator may not be available and it is then useful to be able to calculate the 'long way'.

Problem 9. Multiply 178 by −46

When the numbers have different signs, the result will be negative. (With this in mind, the problem can now be solved by multiplying 178 by 46). Following the procedure of Problem 8 gives

$$\begin{array}{r} 178 \\ \times \quad 46 \\ \hline 1068 \\ 7120 \\ \hline 8188 \\ \hline \end{array}$$

Thus, $178 \times 46 = 8188$ and $\mathbf{178 \times (-46) = -8188}$

Problem 10. Determine 1834 ÷ 7

$$\begin{array}{r} 262 \\ 7\overline{)1834} \end{array}$$

(i) 7 into 18 goes 2, remainder 4. Place the 2 above the 8 of 1834 and carry the 4 remainder to the next digit on the right, making it 43

(ii) 7 into 43 goes 6, remainder 1. Place the 6 above the 3 of 1834 and carry the 1 remainder to the next digit on the right, making it 14

(iii) 7 into 14 goes 2, remainder 0. Place 2 above the 4 of 1834

Hence, $1834 \div 7 = 1834/7 = \dfrac{\mathbf{1834}}{\mathbf{7}} = 262$

The method shown is called **short division**.

Problem 11. Determine $5796 \div 12$

$$\begin{array}{r}
483 \\
12\overline{)5796} \\
\underline{48} \\
99 \\
\underline{96} \\
36 \\
\underline{36} \\
\underline{00}
\end{array}$$

(i) 12 into 5 won't go. 12 into 57 goes 4; place 4 above the 7 of 5796

(ii) $4 \times 12 = 48$; place the 48 below the 57 of 5796

(iii) $57 - 48 = 9$

(iv) Bring down the 9 of 5796 to give 99

(v) 12 into 99 goes 8; place 8 above the 9 of 5796

(vi) $8 \times 12 = 96$; place 96 below the 99

(vii) $99 - 96 = 3$

(viii) Bring down the 6 of 5796 to give 36

(ix) 12 into 36 goes 3 exactly.

(x) Place the 3 above the final 6

(xi) $3 \times 12 = 36$; Place the 36 below the 36

(xii) $36 - 36 = 0$

Hence, $5796 \div 12 = 5796/12 = \dfrac{\mathbf{5796}}{\mathbf{12}} = 483$

The method shown is called **long division**.

Now try the following Practice Exercise

Practice Exercise 2 Further problems on multiplication and division (answers on page 422)

Determine the values of the expressions given in Problems 1 to 9, without using a calculator.

1. (a) 78×6 (b) 124×7
2. (a) £261 × 7 (b) £462 × 9
3. (a) 783 kg × 11 (b) 73 kg × 8
4. (a) 27 mm × 13 (b) 77 mm × 12
5. (a) 448×23 (b) $143 \times (-31)$
6. (a) 288 m ÷ 6 (b) 979 m ÷ 11
7. (a) $\dfrac{1813}{7}$ (b) $\dfrac{896}{16}$
8. (a) $\dfrac{21424}{13}$ (b) $15900 \div -15$
9. (a) $\dfrac{88737}{11}$ (b) $46858 \div 14$
10. A screw has a mass of 15 grams. Calculate, in kilograms, the mass of 1200 such screws (1 kg = 1000 g).
11. Holes are drilled 35.7 mm apart in a metal plate. If a row of 26 holes is drilled, determine the distance, in centimetres, between the centres of the first and last holes.
12. A builder needs to clear a site of bricks and top soil. The total weight to be removed is 696 tonnes. Trucks can carry a maximum load of 24 tonnes. Determine the number of truck loads needed to clear the site.

1.4 Highest common factors and lowest common multiples

When two or more numbers are multiplied together, the individual numbers are called **factors**. Thus, a factor is a number which divides into another number exactly. The **highest common factor (HCF)** is the largest number which divides into two or more numbers exactly.

For example, consider the numbers 12 and 15

The factors of 12 are 1, 2, 3, 4, 6 and 12 (i.e. all the numbers that divide into 12).

The factors of 15 are 1, 3, 5 and 15 (i.e. all the numbers that divide into 15).

1 and 3 are the only **common factors**; i.e. numbers which are factors of **both** 12 and 15

Hence, **the HCF of 12 and 15 is 3** since 3 is the highest number which divides into **both** 12 and 15

A **multiple** is a number which contains another number an exact number of times. The smallest number which is exactly divisible by each of two or more numbers is called the **lowest common multiple (LCM)**.

For example, the multiples of 12 are 12, 24, 36, 48, 60, 72, ... and the multiples of 15 are 15, 30, 45, 60, 75, ...

60 is a common multiple (i.e. a multiple of **both** 12 and 15) and there are no lower common multiples.

Hence, **the LCM of 12 and 15 is 60** since 60 is the lowest number that both 12 and 15 divide into.

Here are some further problems involving the determination of HCFs and LCMs.

Problem 12. Determine the HCF of the numbers 12, 30 and 42

Probably the simplest way of determining an HCF is to express each number in terms of its lowest factors. This is achieved by repeatedly dividing by the prime numbers 2, 3, 5, 7, 11, 13, . . . (where possible) in turn. Thus,

$$\begin{aligned}
12 &= \boxed{2} \times 2 \times \boxed{3} \\
30 &= \boxed{2} \times \boxed{3} \times 5 \\
42 &= \boxed{2} \times \boxed{3} \times 7
\end{aligned}$$

The factors which are common to each of the numbers are 2 in column 1 and 3 in column 3, shown by the broken lines. Hence, **the HCF is 2 × 3**; i.e. **6**. That is, 6 is the largest number which will divide into 12, 30 and 42.

Problem 13. Determine the HCF of the numbers 30, 105, 210 and 1155

Using the method shown in Problem 12:

$$\begin{aligned}
30 &= 2 \times \boxed{3} \times \boxed{5} \\
105 &= \boxed{3} \times \boxed{5} \times 7 \\
210 &= 2 \times \boxed{3} \times \boxed{5} \times 7 \\
1155 &= \boxed{3} \times \boxed{5} \times 7 \times 11
\end{aligned}$$

The factors which are common to each of the numbers are 3 in column 2 and 5 in column 3. Hence, **the HCF is 3 × 5 = 15**

Problem 14. Determine the LCM of the numbers 12, 42 and 90

The LCM is obtained by finding the lowest factors of each of the numbers, as shown in Problems 12 and 13 above, and then selecting the largest group of any of the factors present. Thus,

$$\begin{aligned}
12 &= \boxed{2 \times 2} \times 3 \\
42 &= 2 \times 3 \times \boxed{7} \\
90 &= 2 \times \boxed{3 \times 3} \times \boxed{5}
\end{aligned}$$

The largest group of any of the factors present is shown by the broken lines and is 2 × 2 in 12, 3 × 3 in 90, 5 in 90 and 7 in 42

Hence, **the LCM is 2 x 2 x 3 x 3 x 5 x 7 = 1260** and is the smallest number which 12, 42 and 90 will all divide into exactly.

Problem 15. Determine the LCM of the numbers 150, 210, 735 and 1365

Using the method shown in Problem 14 above:

$$\begin{aligned}
150 &= \boxed{2} \times \boxed{3} \times \boxed{5 \times 5} \\
210 &= 2 \times 3 \times 5 \times 7 \\
735 &= 3 \times 5 \times \boxed{7 \times 7} \\
1365 &= 3 \times 5 \times 7 \times \boxed{13}
\end{aligned}$$

Hence, **the LCM is 2 x 3 x 5 x 5 x 7 x 7 x 13 = 95550**

Now try the following Practice Exercise

Practice Exercise 3 Further problems on highest common factors and lowest common multiples (answers on page 422)

Find (a) the HCF and (b) the LCM of the following groups of numbers.

1. 8, 12
2. 60, 72
3. 50, 70
4. 270, 900
5. 6, 10, 14
6. 12, 30, 45

7. 10, 15, 70, 105
8. 90, 105, 300
9. 196, 210, 462, 910
10. 196, 350, 770

1.5 Order of operation and brackets

1.5.1 Order of operation

Sometimes addition, subtraction, multiplication, division, powers and brackets may all be involved in a calculation. For example,

$$5-3\times 4+24\div(3+5)-3^2$$

This is an extreme example but will demonstrate the order that is necessary when evaluating.
When we read, we read from left to right. However, with mathematics there is a definite order of precedence which we need to adhere to. The order is as follows:

Brackets
Order (or p**O**wer)
Division
Multiplication
Addition
Subtraction

Notice that the first letters of each word spell **BODMAS**, a handy aide-mémoire. **O**rder means p**O**wer. For example, $4^2=4\times 4=16$
$5-3\times 4+24\div(3+5)-3^2$ is evaluated as follows:

$$\begin{aligned}
&5-3\times 4+24\div(3+5)-3^2\\
&\quad =5-3\times 4+24\div 8-3^2 && (\textbf{B}\text{racket is removed and } 3+5 \text{ replaced with } 8)\\
&\quad =5-3\times 4+24\div 8-9 && (\textbf{O}\text{rder means p}\textbf{O}\text{wer; in this case, } 3^2=3\times 3=9)\\
&\quad =5-3\times 4+3-9 && (\textbf{D}\text{ivision: } 24\div 8=3)\\
&\quad =5-12+3-9 && (\textbf{M}\text{ultiplication: } -3\times 4=-12)\\
&\quad =8-12-9 && (\textbf{A}\text{ddition: } 5+3=8)\\
&\quad =\mathbf{-13} && (\textbf{S}\text{ubtraction: } 8-12-9=-13)
\end{aligned}$$

In practice, **it does not matter if multiplication is performed before division or if subtraction is performed before addition**. What is important is that **the process of multiplication and division must be completed before addition and subtraction**.

1.5.2 Brackets and operators

The basic laws governing the **use of brackets and operators** are shown by the following examples.

(a) $2+3=3+2$; i.e. the order of numbers when adding does not matter.

(b) $2\times 3=3\times 2$; i.e. the order of numbers when multiplying does not matter.

(c) $2+(3+4)=(2+3)+4$; i.e. the use of brackets when adding does not affect the result.

(d) $2\times(3\times 4)=(2\times 3)\times 4$; i.e. the use of brackets when multiplying does not affect the result.

(e) $2\times(3+4)=2(3+4)=2\times 3+2\times 4$; i.e. a number placed outside of a bracket indicates that the whole contents of the bracket must be multiplied by that number.

(f) $(2+3)(4+5)=(5)(9)=5\times 9=45$; i.e. adjacent brackets indicate multiplication.

(g) $2[3+(4\times 5)]=2[3+20]=2\times 23=46$; i.e. when an expression contains inner and outer brackets, **the inner brackets are removed first**.

Here are some further problems in which BODMAS needs to be used.

Problem 16. Find the value of $6+4\div(5-3)$

The order of precedence of operations is remembered by the word BODMAS. Thus,

$$\begin{aligned}
6+4\div(5-3)&=6+4\div 2 && (\textbf{B}\text{rackets})\\
&=6+2 && (\textbf{D}\text{ivision})\\
&=\mathbf{8} && (\textbf{A}\text{ddition})
\end{aligned}$$

Problem 17. Determine the value of $13-2\times 3+14\div(2+5)$

$$\begin{aligned}
13-2\times 3+14\div(2+5)&=13-2\times 3+14\div 7 && \text{(B)}\\
&=13-2\times 3+2 && \text{(D)}\\
&=13-6+2 && \text{(M)}\\
&=15-6 && \text{(A)}\\
&=\mathbf{9} && \text{(S)}
\end{aligned}$$

Problem 18. Evaluate $16 \div (2+6) + 18[3 + (4 \times 6) - 21]$

$$\begin{aligned} &16 \div (2+6) + 18[3+(4\times 6)-21] \\ &\quad = 16 \div (2+6) + 18[3+24-21] \quad \text{(B: inner bracket is determined first)} \\ &\quad = 16 \div 8 + 18 \times 6 \quad \text{(B)} \\ &\quad = 2 + 18 \times 6 \quad \text{(D)} \\ &\quad = 2 + 108 \quad \text{(M)} \\ &\quad = \mathbf{110} \quad \text{(A)} \end{aligned}$$

Note that a number outside of a bracket multiplies all that is inside the brackets. In this case,

$$18[3+24-21] = 18[6], \text{ which means } 18 \times 6 = 108$$

Problem 19. Find the value of $23 - 4(2 \times 7) + \dfrac{(144 \div 4)}{(14-8)}$

$$\begin{aligned} 23 - 4(2\times 7) + \frac{(144\div 4)}{(14-8)} &= 23 - 4 \times 14 + \frac{36}{6} \quad \text{(B)} \\ &= 23 - 4 \times 14 + 6 \quad \text{(D)} \\ &= 23 - 56 + 6 \quad \text{(M)} \\ &= 29 - 56 \quad \text{(A)} \\ &= \mathbf{-27} \quad \text{(S)} \end{aligned}$$

Problem 20. Evaluate

$$\frac{3 + \sqrt{(5^2 - 3^2)} + 2^3}{1 + (4 \times 6) \div (3 \times 4)} + \frac{15 \div 3 + 2 \times 7 - 1}{3 \times \sqrt{4} + 8 - 3^2 + 1}$$

$$\begin{aligned} &\frac{3 + \sqrt{(5^2 - 3^2)} + 2^3}{1 + (4 \times 6) \div (3 \times 4)} + \frac{15 \div 3 + 2 \times 7 - 1}{3 \times \sqrt{4} + 8 - 3^2 + 1} \\ &\quad = \frac{3+4+8}{1+24\div 12} + \frac{15 \div 3 + 2 \times 7 - 1}{3 \times 2 + 8 - 9 + 1} \\ &\quad = \frac{3+4+8}{1+2} + \frac{5 + 2 \times 7 - 1}{3 \times 2 + 8 - 9 + 1} \\ &\quad = \frac{15}{3} + \frac{5+14-1}{6+8-9+1} \\ &\quad = 5 + \frac{18}{6} \\ &\quad = 5 + 3 = \mathbf{8} \end{aligned}$$

Now try the following Practice Exercise

Practice Exercise 4 Further problems on order of precedence and brackets (answers on page 422)

Evaluate the following expressions.

1. $14 + 3 \times 15$
2. $17 - 12 \div 4$
3. $86 + 24 \div (14 - 2)$
4. $7(23 - 18) \div (12 - 5)$
5. $63 - 8(14 \div 2) + 26$
6. $\dfrac{40}{5} - 42 \div 6 + (3 \times 7)$
7. $\dfrac{(50-14)}{3} + 7(16-7) - 7$
8. $\dfrac{(7-3)(1-6)}{4(11-6) \div (3-8)}$
9. $\dfrac{(3 + 9 \times 6) \div 3 - 2 \div 2}{3 \times 6 + (4-9) - 3^2 + 5}$
10. $\dfrac{(4 \times 3^2 + 24) \div 5 + 9 \times 3}{2 \times 3^2 - 15 \div 3} + \dfrac{2 + 27 \div 3 + 12 \div 2 - 3^2}{5 + (13 - 2 \times 5) - 4}$
11. $\dfrac{1 + \sqrt{25} + 3 \times 2 - 8 \div 2}{3 \times 4 - \sqrt{(3^2 + 4^2)} + 1} - \dfrac{(4 \times 2 + 7 \times 2) \div 11}{\sqrt{9} + 12 \div 2 - 2^3}$

For fully worked solutions to each of the problems in Practice Exercises 5 to 7 in this chapter, go to the website:

www.routledge.com/cw/bird

Chapter 2

Fractions

***Why it is important to understand:* Fractions**

Engineers use fractions all the time, examples including stress to strain ratios in mechanical engineering, chemical concentration ratios and reaction rates, and ratios in electrical equations to solve for current and voltage. Fractions are also used everywhere in science, from radioactive decay rates to statistical analysis. Calculators are able to handle calculations with fractions. However, there will be times when a quick calculation involving addition, subtraction, multiplication and division of fractions is needed. Again, do not spend too much time on this chapter because we deal with the calculator later; however, try to have some idea how to do quick calculations in the absence of a calculator. You will feel more confident to deal with fractions and calculations if you can do this.

At the end of this chapter, you should be able to:

- understand the terminology numerator, denominator, proper and improper fractions and mixed numbers
- add and subtract fractions
- multiply and divide two fractions
- appreciate the order of operation when evaluating expressions involving fractions

2.1 Introduction

A mark of 9 out of 14 in an examination may be written as $\frac{9}{14}$ or 9/14. $\frac{9}{14}$ is an example of a fraction. The number above the line, i.e. 9, is called the **numerator**. The number below the line, i.e. 14, is called the **denominator**.

When the value of the numerator is less than the value of the denominator, the fraction is called a **proper fraction**. $\frac{9}{14}$ is an example of a proper fraction.

When the value of the numerator is greater than the value of the denominator, the fraction is called an **improper fraction**. $\frac{5}{2}$ is an example of an improper fraction.

A **mixed number** is a combination of a whole number and a fraction. $2\frac{1}{2}$ is an example of a mixed number. In fact, $\frac{5}{2} = 2\frac{1}{2}$

There are a number of everyday examples in which fractions are readily referred to. For example, three people equally sharing a bar of chocolate would have $\frac{1}{3}$ each. A supermarket advertises $\frac{1}{5}$ off a six-pack of beer; if the beer normally costs £2 then it will now cost £1.60. $\frac{3}{4}$ of the employees of a company are women; if the company has 48 employees, then 36 are women.

Calculators are able to handle calculations with fractions. However, to understand a little more about fractions we will in this chapter show how to add, subtract,

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multiply and divide with fractions without the use of a calculator.

Problem 1. Change the following improper fractions into mixed numbers:

$$\text{(a)}\ \frac{9}{2} \quad \text{(b)}\ \frac{13}{4} \quad \text{(c)}\ \frac{28}{5}$$

(a) $\frac{9}{2}$ means 9 halves and $\frac{9}{2} = 9 \div 2$, and $9 \div 2 = 4$ and 1 half, i.e.

$$\frac{9}{2} = \mathbf{4\frac{1}{2}}$$

(b) $\frac{13}{4}$ means 13 quarters and $\frac{13}{4} = 13 \div 4$, and $13 \div 4 = 3$ and 1 quarter, i.e.

$$\frac{13}{4} = \mathbf{3\frac{1}{4}}$$

(c) $\frac{28}{5}$ means 28 fifths and $\frac{28}{5} = 28 \div 5$, and $28 \div 5 =$ 5 and 3 fifths, i.e.

$$\frac{28}{5} = \mathbf{5\frac{3}{5}}$$

Problem 2. Change the following mixed numbers into improper fractions:

$$\text{(a)}\ 5\frac{3}{4} \quad \text{(b)}\ 1\frac{7}{9} \quad \text{(c)}\ 2\frac{3}{7}$$

(a) $5\frac{3}{4}$ means $5 + \frac{3}{4}$. 5 contains $5 \times 4 = 20$ quarters.

Thus, $5\frac{3}{4}$ contains $20 + 3 = 23$ quarters, i.e.

$$5\frac{3}{4} = \mathbf{\frac{23}{4}}$$

The quick way to change $5\frac{3}{4}$ into an improper fraction is $\frac{4 \times 5 + 3}{4} = \mathbf{\frac{23}{4}}$

(b) $1\frac{7}{9} = \frac{9 \times 1 + 7}{9} = \mathbf{\frac{16}{9}}$

(c) $2\frac{3}{7} = \frac{7 \times 2 + 3}{7} = \mathbf{\frac{17}{7}}$

Problem 3. In a school there are 180 students of which 72 are girls. Express this as a fraction in its simplest form

The fraction of girls is $\frac{72}{180}$

Dividing both the numerator and denominator by the lowest prime number, i.e. 2, gives

$$\frac{72}{180} = \frac{36}{90}$$

Dividing both the numerator and denominator again by 2 gives

$$\frac{72}{180} = \frac{36}{90} = \frac{18}{45}$$

2 will not divide into both 18 and 45, so dividing both the numerator and denominator by the next prime number, i.e. 3, gives

$$\frac{72}{180} = \frac{36}{90} = \frac{18}{45} = \frac{6}{15}$$

Dividing both the numerator and denominator again by 3 gives

$$\frac{72}{180} = \frac{36}{90} = \frac{18}{45} = \frac{6}{15} = \frac{2}{5}$$

So $\mathbf{\frac{72}{180} = \frac{2}{5}}$ **in its simplest form.**

Thus, $\mathbf{\frac{2}{5}}$ **of the students are girls.**

2.2 Adding and subtracting fractions

When the denominators of two (or more) fractions to be added are the same, the fractions can be added 'on sight'.

For example, $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$ and $\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$

In the latter example, dividing both the 4 and the 8 by 4 gives $\frac{4}{8} = \frac{1}{2}$, which is the simplified answer. This is called **cancelling**.

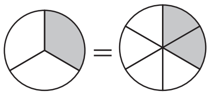
Addition and subtraction of fractions is demonstrated in the following worked examples.

Problem 4. Simplify $\frac{1}{3} + \frac{1}{2}$

(i) Make the denominators the same for each fraction. The lowest number that both denominators divide into is called the **lowest common multiple** or **LCM** (see Chapter 1, page 5). In this example, the LCM of 3 and 2 is 6

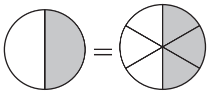
(ii) 3 divides into 6 twice. Multiplying both numerator and denominator of $\frac{1}{3}$ by 2 gives

$$\frac{1}{3} = \frac{2}{6}$$



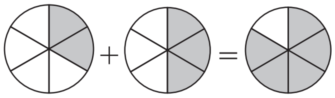
(iii) 2 divides into 6, 3 times. Multiplying both numerator and denominator of $\frac{1}{2}$ by 3 gives

$$\frac{1}{2} = \frac{3}{6}$$



(iv) Hence,

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \mathbf{\frac{5}{6}}$$

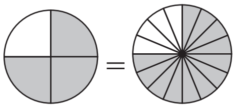


Problem 5. Simplify $\frac{3}{4} - \frac{7}{16}$

(i) Make the denominators the same for each fraction. The lowest common multiple (LCM) of 4 and 16 is 16

(ii) 4 divides into 16, 4 times. Multiplying both numerator and denominator of $\frac{3}{4}$ by 4 gives

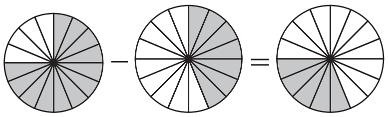
$$\frac{3}{4} = \frac{12}{16}$$



(iii) $\frac{7}{16}$ already has a denominator of 16

(iv) Hence,

$$\frac{3}{4} - \frac{7}{16} = \frac{12}{16} - \frac{7}{16} = \mathbf{\frac{5}{16}}$$



Problem 6. Simplify $4\frac{2}{3} - 1\frac{1}{6}$

$4\frac{2}{3} - 1\frac{1}{6}$ is the same as $\left(4\frac{2}{3}\right) - \left(1\frac{1}{6}\right)$ which is the same as $\left(4 + \frac{2}{3}\right) - \left(1 + \frac{1}{6}\right)$ which is the same as $4 + \frac{2}{3} - 1 - \frac{1}{6}$ which is the same as $3 + \frac{2}{3} - \frac{1}{6}$ which is the same as $3 + \frac{4}{6} - \frac{1}{6} = 3 + \frac{3}{6} = 3 + \frac{1}{2}$

Thus, $4\frac{2}{3} - 1\frac{1}{6} = \mathbf{3\frac{1}{2}}$

Problem 7. Evaluate $7\frac{1}{8} - 5\frac{3}{7}$

$$\begin{aligned}
7\frac{1}{8} - 5\frac{3}{7} &= \left(7 + \frac{1}{8}\right) - \left(5 + \frac{3}{7}\right) = 7 + \frac{1}{8} - 5 - \frac{3}{7} \\
&= 2 + \frac{1}{8} - \frac{3}{7} = 2 + \frac{7 \times 1 - 8 \times 3}{56} \\
&= 2 + \frac{7 - 24}{56} = 2 + \frac{-17}{56} = 2 - \frac{17}{56} \\
&= \frac{112}{56} - \frac{17}{56} = \frac{112 - 17}{56} = \frac{95}{56} = \mathbf{1\frac{39}{56}}
\end{aligned}$$

Problem 8. Determine the value of

$$4\frac{5}{8} - 3\frac{1}{4} + 1\frac{2}{5}$$

$$\begin{aligned}
4\frac{5}{8} - 3\frac{1}{4} + 1\frac{2}{5} &= (4 - 3 + 1) + \left(\frac{5}{8} - \frac{1}{4} + \frac{2}{5}\right) \\
&= 2 + \frac{5 \times 5 - 10 \times 1 + 8 \times 2}{40} \\
&= 2 + \frac{25 - 10 + 16}{40} \\
&= 2 + \frac{31}{40} = \mathbf{2\frac{31}{40}}
\end{aligned}$$

Now try the following Practice Exercise

Practice Exercise 5 Introduction to fractions (answers on page 422)

1. Change the improper fraction $\frac{15}{7}$ into a mixed number.
2. Change the improper fraction $\frac{37}{5}$ into a mixed number.
3. Change the mixed number $2\frac{4}{9}$ into an improper fraction.

4. Change the mixed number $8\frac{7}{8}$ into an improper fraction.

5. A box contains 165 paper clips. 60 clips are removed from the box. Express this as a fraction in its simplest form.

6. Order the following fractions from the smallest to the largest.

$$\frac{4}{9}, \frac{5}{8}, \frac{3}{7}, \frac{1}{2}, \frac{3}{5}$$

7. A training college has 375 students of which 120 are girls. Express this as a fraction in its simplest form.

Evaluate, in fraction form, the expressions given in Problems 8 to 20.

8. $\frac{1}{3}+\frac{2}{5}$
9. $\frac{5}{6}-\frac{4}{15}$
10. $\frac{1}{2}+\frac{2}{5}$
11. $\frac{7}{16}-\frac{1}{4}$
12. $\frac{2}{7}+\frac{3}{11}$
13. $\frac{2}{9}-\frac{1}{7}+\frac{2}{3}$
14. $3\frac{2}{5}-2\frac{1}{3}$
15. $\frac{7}{27}-\frac{2}{3}+\frac{5}{9}$
16. $5\frac{3}{13}+3\frac{3}{4}$
17. $4\frac{5}{8}-3\frac{2}{5}$
18. $10\frac{3}{7}-8\frac{2}{3}$
19. $3\frac{1}{4}-4\frac{4}{5}+1\frac{5}{6}$
20. $5\frac{3}{4}-1\frac{2}{5}-3\frac{1}{2}$

2.3 Multiplication and division of fractions

2.3.1 Multiplication

To multiply two or more fractions together, the numerators are first multiplied to give a single number and this becomes the new numerator of the combined fraction. The denominators are then multiplied together to give the new denominator of the combined fraction.

For example, $\frac{2}{3}\times\frac{4}{7}=\frac{2\times 4}{3\times 7}=\mathbf{\frac{8}{21}}$

Problem 9. Simplify $7\times\frac{2}{5}$

$$7\times\frac{2}{5}=\frac{7}{1}\times\frac{2}{5}=\frac{7\times 2}{1\times 5}=\frac{14}{5}=\mathbf{2\frac{4}{5}}$$

Problem 10. Find the value of $\frac{3}{7}\times\frac{14}{15}$

Dividing numerator and denominator by 3 gives

$$\frac{3}{7}\times\frac{14}{15}=\frac{1}{7}\times\frac{14}{5}=\frac{1\times 14}{7\times 5}$$

Dividing numerator and denominator by 7 gives

$$\frac{1\times 14}{7\times 5}=\frac{1\times 2}{1\times 5}=\mathbf{\frac{2}{5}}$$

This process of dividing both the numerator and denominator of a fraction by the same factor(s) is called **cancelling**.

Problem 11. Simplify $\frac{3}{5}\times\frac{4}{9}$

$$\frac{3}{5}\times\frac{4}{9}=\frac{1}{5}\times\frac{4}{3} \text{ by cancelling}$$
$$=\mathbf{\frac{4}{15}}$$

Problem 12. Evaluate $1\frac{3}{5}\times 2\frac{1}{3}\times 3\frac{3}{7}$

Mixed numbers **must** be expressed as improper fractions before multiplication can be performed. Thus,

$$1\frac{3}{5}\times 2\frac{1}{3}\times 3\frac{3}{7}=\left(\frac{5}{5}+\frac{3}{5}\right)\times\left(\frac{6}{3}+\frac{1}{3}\right)\times\left(\frac{21}{7}+\frac{3}{7}\right)$$
$$=\frac{8}{5}\times\frac{7}{3}\times\frac{24}{7}=\frac{8\times 1\times 8}{5\times 1\times 1}=\frac{64}{5}$$
$$=\mathbf{12\frac{4}{5}}$$

Problem 13. Simplify $3\frac{1}{5}\times 1\frac{2}{3}\times 2\frac{3}{4}$

The mixed numbers need to be changed to improper fractions before multiplication can be performed.

$$3\frac{1}{5}\times 1\frac{2}{3}\times 2\frac{3}{4}=\frac{16}{5}\times\frac{5}{3}\times\frac{11}{4}$$

$$=\frac{4}{1}\times\frac{1}{3}\times\frac{11}{1}\text{ by cancelling}$$

$$=\frac{4\times 1\times 11}{1\times 3\times 1}=\frac{44}{3}=\mathbf{14\frac{2}{3}}$$

2.3.2 Division

The simple rule for division is **change the division sign into a multiplication sign and invert the second fraction**.

For example, $\frac{2}{3}\div\frac{3}{4}=\frac{2}{3}\times\frac{4}{3}=\mathbf{\frac{8}{9}}$

Problem 14. Simplify $\frac{3}{7}\div\frac{8}{21}$

$$\frac{3}{7}\div\frac{8}{21}=\frac{3}{7}\times\frac{21}{8}=\frac{3}{1}\times\frac{3}{8}\text{ by cancelling}$$

$$=\frac{3\times 3}{1\times 8}=\frac{9}{8}=\mathbf{1\frac{1}{8}}$$

Problem 15. Find the value of $5\frac{3}{5}\div 7\frac{1}{3}$

The mixed numbers must be expressed as improper fractions. Thus,

$$5\frac{3}{5}\div 7\frac{1}{3}=\frac{28}{5}\div\frac{22}{3}=\frac{28}{5}\times\frac{3}{22}=\frac{14}{5}\times\frac{3}{11}=\mathbf{\frac{42}{55}}$$

Problem 16. Simplify $3\frac{2}{3}\times 1\frac{3}{4}\div 2\frac{3}{4}$

Mixed numbers must be expressed as improper fractions before multiplication and division can be performed:

$$3\frac{2}{3}\times 1\frac{3}{4}\div 2\frac{3}{4}=\frac{11}{3}\times\frac{7}{4}\div\frac{11}{4}=\frac{11}{3}\times\frac{7}{4}\times\frac{4}{11}$$

$$=\frac{1\times 7\times 1}{3\times 1\times 1}\text{ by cancelling}$$

$$=\frac{7}{3}=\mathbf{2\frac{1}{3}}$$

Now try the following Practice Exercise

Practice Exercise 6 Multiplying and dividing fractions (answers on page 422)

Evaluate the following.

1. $\frac{2}{5}\times\frac{4}{7}$
2. $5\times\frac{4}{9}$
3. $\frac{3}{4}\times\frac{8}{11}$
4. $\frac{3}{4}\times\frac{5}{9}$
5. $\frac{17}{35}\times\frac{15}{68}$
6. $\frac{3}{5}\times\frac{7}{9}\times 1\frac{2}{7}$
7. $\frac{13}{17}\times 4\frac{7}{11}\times 3\frac{4}{39}$
8. $\frac{1}{4}\times\frac{3}{11}\times 1\frac{5}{39}$
9. $\frac{2}{9}\div\frac{4}{27}$
10. $\frac{3}{8}\div\frac{45}{64}$
11. $\frac{3}{8}\div\frac{5}{32}$
12. $\frac{3}{4}\div 1\frac{4}{5}$
13. $2\frac{1}{4}\times 1\frac{2}{3}$
14. $1\frac{1}{3}\div 2\frac{5}{9}$
15. $2\frac{4}{5}\div\frac{7}{10}$
16. $2\frac{3}{4}\div 3\frac{2}{3}$
17. $\frac{1}{9}\times\frac{3}{4}\times 1\frac{1}{3}$
18. $3\frac{1}{4}\times 1\frac{3}{5}\div\frac{2}{5}$
19. A ship's crew numbers 105, of which $\frac{1}{7}$ are women. Of the men, $\frac{1}{6}$ are officers. How many male officers are on board?
20. If a storage tank is holding 450 litres when it is three-quarters full, how much will it contain when it is two-thirds full?
21. Three people, P, Q and R, contribute to a fund. P provides 3/5 of the total, Q provides 2/3 of the remainder and R provides £8. Determine (a) the total of the fund and (b) the contributions of P and Q.
22. A tank contains 24,000 litres of oil. Initially, $\frac{7}{10}$ of the contents are removed, then $\frac{3}{5}$ of the remainder is removed. How much oil is left in the tank?

2.4 Order of operation with fractions

As stated in Chapter 1, sometimes addition, subtraction, multiplication, division, powers and brackets can all be involved in a calculation. A definite order of precedence must be adhered to. The order is:

Brackets

Order (or p**O**wer)

Division

Multiplication

Addition

Subtraction

This is demonstrated in the following worked problems.

Problem 17. Simplify $\frac{7}{20}-\frac{3}{8}\times\frac{4}{5}$

$$\begin{aligned}
\frac{7}{20}-\frac{3}{8}\times\frac{4}{5} &= \frac{7}{20}-\frac{3\times 1}{2\times 5} \text{ by cancelling} \\
&= \frac{7}{20}-\frac{3}{10} && \text{(M)} \\
&= \frac{7}{20}-\frac{6}{20} \\
&= \mathbf{\frac{1}{20}} && \text{(S)}
\end{aligned}$$

Problem 18. Simplify $\frac{1}{4}-2\frac{1}{5}\times\frac{5}{8}+\frac{9}{10}$

$$\begin{aligned}
\frac{1}{4}-2\frac{1}{5}\times\frac{5}{8}+\frac{9}{10} &= \frac{1}{4}-\frac{11}{5}\times\frac{5}{8}+\frac{9}{10} \\
&= \frac{1}{4}-\frac{11}{1}\times\frac{1}{8}+\frac{9}{10} \text{ by cancelling} \\
&= \frac{1}{4}-\frac{11}{8}+\frac{9}{10} && \text{(M)} \\
&= \frac{1\times 10}{4\times 10}-\frac{11\times 5}{8\times 5}+\frac{9\times 4}{10\times 4} \\
&\text{(since the LCM of 4, 8 and 10 is 40)} \\
&= \frac{10}{40}-\frac{55}{40}+\frac{36}{40} \\
&= \frac{10-55+36}{40} && \text{(A/S)} \\
&= -\mathbf{\frac{9}{40}}
\end{aligned}$$

Problem 19. Simplify

$$2\frac{1}{2}-\left(\frac{2}{5}+\frac{3}{4}\right)\div\left(\frac{5}{8}\times\frac{2}{3}\right)$$

$$\begin{aligned}
&2\frac{1}{2}-\left(\frac{2}{5}+\frac{3}{4}\right)\div\left(\frac{5}{8}\times\frac{2}{3}\right) \\
&= \frac{5}{2}-\left(\frac{2\times 4}{5\times 4}+\frac{3\times 5}{4\times 5}\right)\div\left(\frac{5}{8}\times\frac{2}{3}\right) && \text{(B)} \\
&= \frac{5}{2}-\left(\frac{8}{20}+\frac{15}{20}\right)\div\left(\frac{5}{8}\times\frac{2}{3}\right) && \text{(B)} \\
&= \frac{5}{2}-\frac{23}{20}\div\left(\frac{5}{4}\times\frac{1}{3}\right) \text{ by cancelling} && \text{(B)} \\
&= \frac{5}{2}-\frac{23}{20}\div\frac{5}{12} && \text{(B)} \\
&= \frac{5}{2}-\frac{23}{20}\times\frac{12}{5} && \text{(D)} \\
&= \frac{5}{2}-\frac{23}{5}\times\frac{3}{5} \text{ by cancelling} \\
&= \frac{5}{2}-\frac{69}{25} && \text{(M)} \\
&= \frac{5\times 25}{2\times 25}-\frac{69\times 2}{25\times 2} && \text{(S)} \\
&= \frac{125}{50}-\frac{138}{50} && \text{(S)} \\
&= -\mathbf{\frac{13}{50}}
\end{aligned}$$

Problem 20. Evaluate

$$\frac{1}{3}\text{ of }\left(5\frac{1}{2}-3\frac{3}{4}\right)+3\frac{1}{5}\div\frac{4}{5}-\frac{1}{2}$$

$$\begin{aligned}
&\frac{1}{3}\text{ of }\left(5\frac{1}{2}-3\frac{3}{4}\right)+3\frac{1}{5}\div\frac{4}{5}-\frac{1}{2} \\
&= \frac{1}{3}\text{ of }1\frac{3}{4}+3\frac{1}{5}\div\frac{4}{5}-\frac{1}{2} && \text{(B)} \\
&= \frac{1}{3}\times\frac{7}{4}+\frac{16}{5}\div\frac{4}{5}-\frac{1}{2} && \text{(O)}
\end{aligned}$$

(Note that the 'of' is replaced with a multiplication sign.)

$$\begin{aligned}
&= \frac{1}{3}\times\frac{7}{4}+\frac{16}{5}\times\frac{5}{4}-\frac{1}{2} && \text{(D)} \\
&= \frac{1}{3}\times\frac{7}{4}+\frac{4}{1}\times\frac{1}{1}-\frac{1}{2} \text{ by cancelling}
\end{aligned}$$

$$= \frac{7}{12} + \frac{4}{1} - \frac{1}{2} \qquad \text{(M)}$$

$$= \frac{7}{12} + \frac{48}{12} - \frac{6}{12} \qquad \text{(A/S)}$$

$$= \frac{49}{12}$$

$$= \mathbf{4\frac{1}{12}}$$

Now try the following Practice Exercise

Practice Exercise 7 Order of operation with fractions (answers on page 422)

Evaluate the following.

1. $2\frac{1}{2} - \frac{3}{5} \times \frac{20}{27}$
2. $\frac{1}{3} - \frac{3}{4} \times \frac{16}{27}$
3. $\frac{1}{2} + \frac{3}{5} \div \frac{9}{15} - \frac{1}{3}$
4. $\frac{1}{5} + 2\frac{2}{3} \div \frac{5}{9} - \frac{1}{4}$
5. $\frac{4}{5} \times \frac{1}{2} - \frac{1}{6} \div \frac{2}{5} + \frac{2}{3}$
6. $\frac{3}{5} - \left(\frac{2}{3} - \frac{1}{2}\right) \div \left(\frac{5}{6} \times \frac{3}{2}\right)$
7. $\frac{1}{2}$ of $\left(4\frac{2}{5} - 3\frac{7}{10}\right) + \left(3\frac{1}{3} \div \frac{2}{3}\right) - \frac{2}{5}$
8. $\dfrac{6\frac{2}{3} \times 1\frac{2}{5} - \frac{1}{3}}{6\frac{3}{4} \div 1\frac{1}{2}}$
9. $1\frac{1}{3} \times 2\frac{1}{5} \div \frac{2}{5}$
10. $\frac{1}{4} \times \frac{2}{5} - \frac{1}{5} \div \frac{2}{3} + \frac{4}{15}$
11. $\dfrac{\frac{2}{3} + 3\frac{1}{5} \times 2\frac{1}{2} + 1\frac{1}{3}}{8\frac{1}{3} \div 3\frac{1}{3}}$
12. $\frac{1}{13}$ of $\left(2\frac{9}{10} - 1\frac{3}{5}\right) + \left(2\frac{1}{3} \div \frac{2}{3}\right) - \frac{3}{4}$

For fully worked solutions to each of the problems in Practice Exercises 5 to 7 in this chapter, go to the website:
www.routledge.com/cw/bird

Revision Test 1: Basic arithmetic and fractions

This assignment covers the material contained in Chapters 1 and 2. *The marks available are shown in brackets at the end of each question.*

1. Evaluate
$1009\,\text{cm} - 356\,\text{cm} - 742\,\text{cm} + 94\,\text{cm}$. (3)

2. Determine $£284 \times 9$ (3)

3. Evaluate
(a) $-11239 - (-4732) + 9639$
(b) -164×-12
(c) 367×-19 (6)

4. Calculate (a) $\$153 \div 9$ (b) $1397\,\text{g} \div 11$ (4)

5. A small component has a mass of 27 grams. Calculate the mass, in kilograms, of 750 such components. (3)

6. Find (a) the highest common factor and (b) the lowest common multiple of the following numbers: 15 40 75 120 (7)

Evaluate the expressions in questions 7 to 12.

7. $7 + 20 \div (9 - 5)$ (3)

8. $147 - 21(24 \div 3) + 31$ (3)

9. $40 \div (1 + 4) + 7[8 + (3 \times 8) - 27]$ (5)

10. $\dfrac{(7-3)(2-5)}{3(9-5) \div (2-6)}$ (3)

11. $\dfrac{(7 + 4 \times 5) \div 3 + 6 \div 2}{2 \times 4 + (5 - 8) - 2^2 + 3}$ (5)

12. $\dfrac{(4^2 \times 5 - 8) \div 3 + 9 \times 8}{4 \times 3^2 - 20 \div 5}$ (5)

13. Simplify
(a) $\dfrac{3}{4} - \dfrac{7}{15}$
(b) $1\dfrac{5}{8} - 2\dfrac{1}{3} + 3\dfrac{5}{6}$ (8)

14. A training college has 480 students of which 150 are girls. Express this as a fraction in its simplest form. (2)

15. A tank contains 18 000 litres of oil. Initially, $\dfrac{7}{10}$ of the contents are removed, then $\dfrac{2}{5}$ of the remainder is removed. How much oil is left in the tank? (4)

16. Evaluate
(a) $1\dfrac{7}{9} \times \dfrac{3}{8} \times 3\dfrac{3}{5}$
(b) $6\dfrac{2}{3} \div 1\dfrac{1}{3}$
(c) $1\dfrac{1}{3} \times 2\dfrac{1}{5} \div \dfrac{2}{5}$ (10)

17. Calculate
(a) $\dfrac{1}{4} \times \dfrac{2}{5} - \dfrac{1}{5} \div \dfrac{2}{3} + \dfrac{4}{15}$
(b) $\dfrac{\dfrac{2}{3} + 3\dfrac{1}{5} \times 2\dfrac{1}{2} + 1\dfrac{1}{3}}{8\dfrac{1}{3} \div 3\dfrac{1}{3}}$ (8)

18. Simplify $\left\{\dfrac{1}{13} \text{ of } \left(2\dfrac{9}{10} - 1\dfrac{3}{5}\right)\right\} + \left(2\dfrac{1}{3} \div \dfrac{2}{3}\right) - \dfrac{3}{4}$ (8)

For lecturers/instructors/teachers, fully worked solutions to each of the problems in Revision Test 1, together with a full marking scheme, are available at the website:
www.routledge.com/cw/bird

Chapter 3

Decimals

***Why it is important to understand:* Decimals**

Engineers and scientists use decimal numbers all the time in calculations. Calculators are able to handle calculations with decimals; however, there will be times when a quick calculation involving addition, subtraction, multiplication and division of decimals is needed. Again, do not spend too much time on this chapter because we deal with the calculator later; however, try to have some idea how to do quick calculations involving decimal numbers in the absence of a calculator. You will feel more confident to deal with decimal numbers in calculations if you can do this.

At the end of this chapter, you should be able to:

- convert a decimal number to a fraction and vice-versa
- understand and use significant figures and decimal places in calculations
- add and subtract decimal numbers
- multiply and divide decimal numbers

3.1 Introduction

The decimal system of numbers is based on the digits 0 to 9.

There are a number of everyday occurrences in which we use decimal numbers. For example, a radio is, say, tuned to 107.5 MHz FM; 107.5 is an example of a decimal number.

In a shop, a pair of trainers cost, say, £57.95; 57.95 is another example of a decimal number. 57.95 is a decimal fraction, where a decimal point separates the integer, i.e. 57, from the fractional part, i.e. 0.95

$$\mathbf{57.95 \text{ actually means } (5 \times 10) + (7 \times 1) + \left(9 \times \frac{1}{10}\right) + \left(5 \times \frac{1}{100}\right)}$$

3.2 Converting decimals to fractions and vice-versa

Converting decimals to fractions and vice-versa is demonstrated below with worked examples.

Problem 1. Convert 0.375 to a proper fraction in its simplest form

(i) 0.375 may be written as $\frac{0.375 \times 1000}{1000}$ i.e. $0.375 = \frac{375}{1000}$

(ii) Dividing both numerator and denominator by 5 gives $\frac{375}{1000} = \frac{75}{200}$

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(iii) Dividing both numerator and denominator by 5 again gives $\frac{75}{200}=\frac{15}{40}$

(iv) Dividing both numerator and denominator by 5 again gives $\frac{15}{40}=\mathbf{\frac{3}{8}}$

Since both 3 and 8 are only divisible by 1, we cannot 'cancel' any further, so $\frac{3}{8}$ is the 'simplest form' of the fraction.

Hence, **the decimal fraction 0.375 = $\mathbf{\frac{3}{8}}$ as a proper fraction**.

Problem 2. Convert 3.4375 to a mixed number

Initially, the whole number 3 is ignored.

(i) 0.4375 may be written as $\frac{0.4375\times 10000}{10000}$ i.e. $0.4375=\frac{4375}{10000}$

(ii) Dividing both numerator and denominator by 25 gives $\frac{4375}{10000}=\frac{175}{400}$

(iii) Dividing both numerator and denominator by 5 gives $\frac{175}{400}=\frac{35}{80}$

(iv) Dividing both numerator and denominator by 5 again gives $\frac{35}{80}=\frac{7}{16}$

Since both 5 and 16 are only divisible by 1, we cannot 'cancel' any further, so $\frac{7}{16}$ is the 'lowest form' of the fraction.

(v) Hence, $0.4375=\frac{7}{16}$

Thus, **the decimal fraction 3.4375 = $\mathbf{3\frac{7}{16}}$ as a mixed number**.

Problem 3. Express $\frac{7}{8}$ as a decimal fraction

To convert a proper fraction to a decimal fraction, the numerator is divided by the denominator.

$$\begin{array}{r} 0.8\,7\,5 \\ 8\overline{)7.0\,0\,0} \end{array}$$

(i) 8 into 7 will not go. Place the 0 above the 7

(ii) Place the decimal point above the decimal point of 7.000

(iii) 8 into 70 goes 8, remainder 6. Place the 8 above the first zero after the decimal point and carry the 6 remainder to the next digit on the right, making it 60

(iv) 8 into 60 goes 7, remainder 4. Place the 7 above the next zero and carry the 4 remainder to the next digit on the right, making it 40

(v) 8 into 40 goes 5, remainder 0. Place 5 above the next zero.

Hence, **the proper fraction $\mathbf{\frac{7}{8}}$ = 0.875 as a decimal fraction**.

Problem 4. Express $5\frac{13}{16}$ as a decimal fraction

For mixed numbers it is only necessary to convert the proper fraction part of the mixed number to a decimal fraction.

$$\begin{array}{r} 0.8\,1\,2\,5 \\ 16\overline{)13.0\,0\,0\,0} \end{array}$$

(i) 16 into 13 will not go. Place the 0 above the 3

(ii) Place the decimal point above the decimal point of 13.0000

(iii) 16 into 130 goes 8, remainder 2. Place the 8 above the first zero after the decimal point and carry the 2 remainder to the next digit on the right, making it 20

(iv) 16 into 20 goes 1, remainder 4. Place the 1 above the next zero and carry the 4 remainder to the next digit on the right, making it 40

(v) 16 into 40 goes 2, remainder 8. Place the 2 above the next zero and carry the 8 remainder to the next digit on the right, making it 80

(vi) 16 into 80 goes 5, remainder 0. Place the 5 above the next zero.

(vii) Hence, $\frac{13}{16}=0.8125$

Thus, **the mixed number $\mathbf{5\frac{13}{16}}$ = 5.8125 as a decimal fraction**.

Now try the following Practice Exercise

Practice Exercise 8 Converting decimals to fractions and vice-versa (answers on page 423)

1. Convert 0.65 to a proper fraction.
2. Convert 0.036 to a proper fraction.
3. Convert 0.175 to a proper fraction.
4. Convert 0.048 to a proper fraction.
5. Convert the following to proper fractions.
   (a) 0.66 (b) 0.84 (c) 0.0125
   (d) 0.282 (e) 0.024
6. Convert 4.525 to a mixed number.
7. Convert 23.44 to a mixed number.
8. Convert 10.015 to a mixed number.
9. Convert 6.4375 to a mixed number.
10. Convert the following to mixed numbers.
    (a) 1.82 (b) 4.275 (c) 14.125
    (d) 15.35 (e) 16.2125
11. Express $\frac{5}{8}$ as a decimal fraction.
12. Express $6\frac{11}{16}$ as a decimal fraction.
13. Express $\frac{7}{32}$ as a decimal fraction.
14. Express $11\frac{3}{16}$ as a decimal fraction.
15. Express $\frac{9}{32}$ as a decimal fraction.

3.3 Significant figures and decimal places

A number which can be expressed exactly as a decimal fraction is called a **terminating decimal**. For example,

$$3\frac{3}{16} = 3.1825 \text{ is a terminating decimal}$$

A number which cannot be expressed exactly as a decimal fraction is called a **non-terminating decimal**. For example,

$$1\frac{5}{7} = 1.7142857\ldots \text{ is a non-terminating decimal}$$

The answer to a non-terminating decimal may be expressed in two ways, depending on the accuracy required:

(a) correct to a number of **significant figures**, or

(b) correct to a number of **decimal places** i.e. the number of figures after the decimal point.

The last digit in the answer is unaltered if the next digit on the right is in the group of numbers 0, 1, 2, 3 or 4. For example,

$$\begin{aligned}1.714285\ldots &= \mathbf{1.714} \text{ correct to 4 significant figures}\\ &= \mathbf{1.714} \text{ correct to 3 decimal places}\end{aligned}$$

since the next digit on the right in this example is 2
The last digit in the answer is increased by 1 if the next digit on the right is in the group of numbers 5, 6, 7, 8 or 9. For example,

$$\begin{aligned}1.7142857\ldots &= \mathbf{1.7143} \text{ correct to 5 significant figures}\\ &= \mathbf{1.7143} \text{ correct to 4 decimal places}\end{aligned}$$

since the next digit on the right in this example is 8

Problem 5. Express 15.36815 correct to
(a) 2 decimal places, (b) 3 significant figures,
(c) 3 decimal places, (d) 6 significant figures

(a) 15.36815 = **15.37** correct to 2 decimal places.

(b) 15.36815 = **15.4** correct to 3 significant figures.

(c) 15.36815 = **15.368** correct to 3 decimal places.

(d) 15.36815 = **15.3682** correct to 6 significant figures.

Problem 6. Express 0.004369 correct to
(a) 4 decimal places, (b) 3 significant figures

(a) 0.004369 = **0.0044** correct to 4 decimal places.

(b) 0.004369 = **0.00437** correct to 3 significant figures.

Note that the zeros to the right of the decimal point do not count as significant figures.